SLEdge: Bounded Model Checking in Separation Logic

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Accurate symbolic execution for C++ via LLVM

- First focus: “memory safety” bugs
  - object lifetime / use-after-free, in various manifestations
- It is ok for a pointer to outlive the object to which it points,
- But it is not ok to dereference it
  - Must hold of all code, no matter what idioms it follows
- Goal is to refute safety properties
- Proving properties is currently a non-goal
  - general-purpose automatic heap abstractions are extremely hard
  - most practical value is in actionable bug reports
  - there are few bugs that cannot be found with a small number of objects

Accurate symbolic execution for C++ via LLVM

Why/how accurate?

- Important to have reasonable understanding of source code
- Need byte-sizes of memory accesses
- Need aliasing between different names of same field (the first member, inline structs, union members, etc.)
- Need relationship between bytewise (e.g. memcpy) and structured (e.g. field access) operations
- Aliasing at byte granularity

Accurate symbolic execution for C++ via *LLVM*

Why LLVM?

- C++: what does it even mean?
- LLVM: much smaller language
  - with semantics closer to level needed by analyzer
- Leave understanding the dark corners of C++ to clang
- Reuse bitcode already produced by standard builds
- Analyze (bit)code close to what runs in production
  - don't have to trust much of clang
  - not analyzing binary: still must trust some of llvm / linker / etc.

Accurate *symbolic* execution for C++ via LLVM

Why symbolic?

- Key for good coverage without good test suite / input generator
- Analysis "executes" all code branches
- Breadth-first exploration, with iterative deepening
- Locality-of-reference a la in hardware but in symbolic representation
- Join execution paths eagerly: DAGify the execution tree
- Enable later adaptation of Infers techniques for scalability
  - Summarization: intermediate results caching
  - Compositionality: "start anywhere" analysis

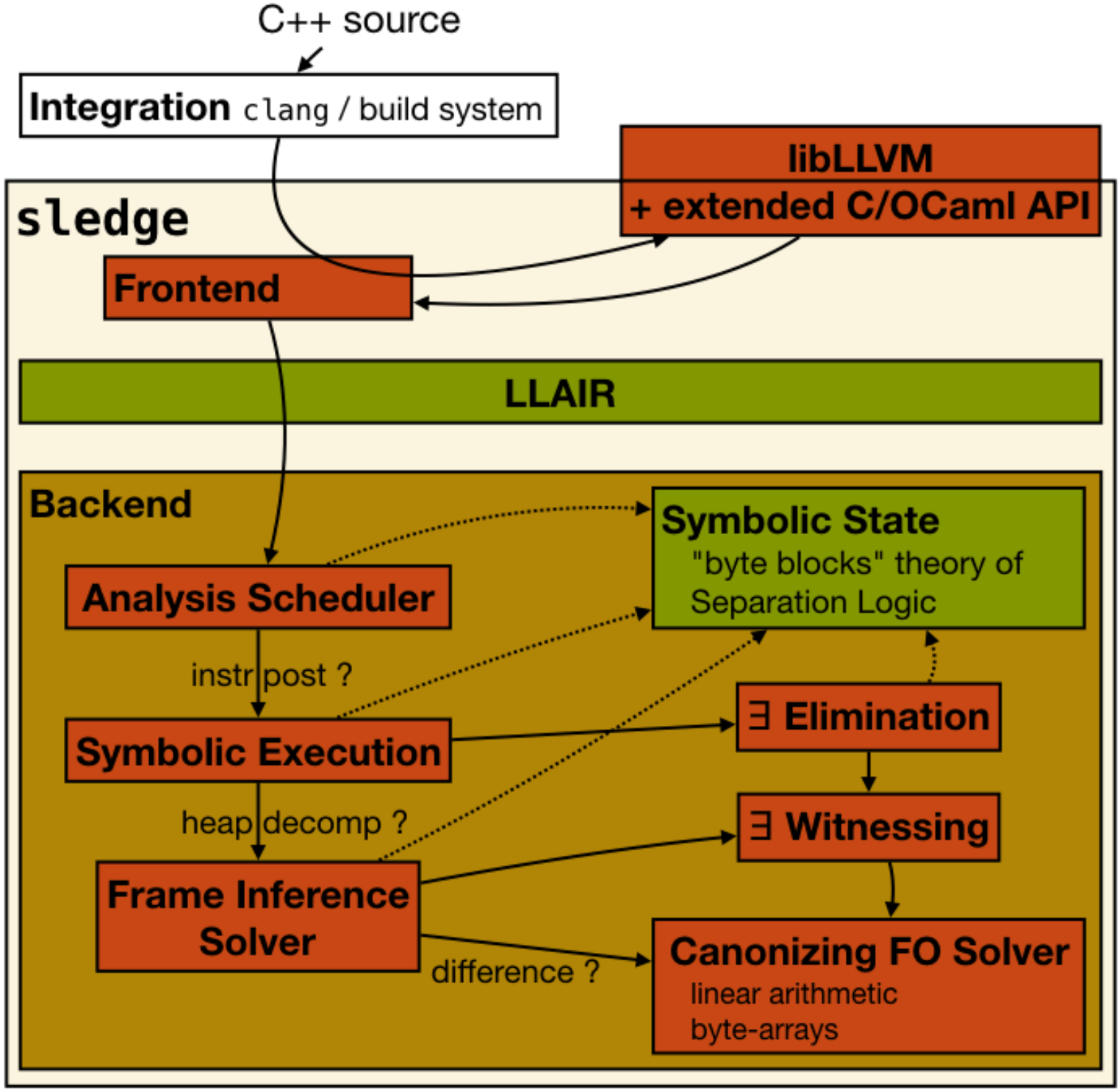
Accurate symbolic execution for *C++* via LLVM

Why C++?

- There is a *lot* of it
- There is a lot of it that matters
- Language is unsafe...
- But does have a justifiable reason to exist (other than legacy)
- Not so far from "generic unsafe code" via LLVM

Current limitations / sacrifices for initial iterations

- Summarization: currently intraprocedural, "as if inlined"
- Compositionality / (bi)abduction: no "start anywhere" capability, doesn't scale as $O(|\text{diff}|)$
- Dynamically-sized arrays imprecisely handled
- Over-approximate: maybe better to prove existence of bugs rather than fail to prove their absence
- C++ exceptions: frontend translation needs to be extended
- ...


C++ source
Integration clang / build system
libLLVM
+ extended C/OCaml API
sledge
Frontend
LLAIR
Backend
Symbolic State
"byte blocks" theory of
Separation Logic
Analysis Scheduler
instr post ?
Symbolic Execution
∃ Elimination
heap decomp ?
∃ Witnessing
Frame Inference
Solver
difference ?
Canonizing FO Solver
linear arithmetic
byte-arrays

LLAIR (Low-Level Analysis Internal Representation)

LLAIR vs. LLVM: different design constraints from different usage

- LLVM: compiler internal representation (IR) aimed at code transformation, generation, etc.
- LLAIR: IR tailored for static analysis using a low-level model of memory

Llair: SSA

Local variables

- LLVM: SSA
  - useful when performing code transformations, especially directed by static analysis results
- LLAIR: parallel-assignment (+ trampolines)
  - *static* single assignment does not buy much as there will be multiple *dynamic* assignments in any case, due to e.g. loops
  - traditional ϕ-nodes need some execution history to interpret: poor fit for analysis operating on representations of sets of states at program points
  - parallel-assignment hard for a compiler, easy for an analyzer
  - SSA essentially in continuation-passing style
  - Hoare logic (with triples) in direct style
  - mismatch: no SSA scope for variables in postconditions representing return values

LLAIR: ALU

Arithmetic and logic operations

- LLVM: linear 3-address code

```
%r1 = add %r2, %r3
%r5 = add %r1, %r4
%r6 = add %r5, %r7
```

- LLAIR: compound expressions over registers

```
%r6 = ((%r2 + %r3) + %r4) + %r7
```

Bounded vs mathematical integers

- LLVM: types have bitwidths, operations have signedness
- LLAIR: backend operates over mathematical integers, frontend adds conversions

LLAIR: Formalization

Formalization in HOL4 by Scott Owens

- semantics of LLVM
- semantics of LLAIR
- model of the translation implemented by the frontend
- correctness proofs
  - (full disclosure: mechanically verified with some unproven assumptions, some simplifications of LLVM e.g. pointers are just integers)
- aim: ensure that bogus/missed bugs are due to analysis trade-offs, not correctly analyzing incorrect code

Analysis Scheduler

- interprets control flow
  - lifts semantics of instructions to whole programs
  - handles scoping and parameter passing
- overall objective: find violations of safety properties using short executions before exploring longer executions
- which of the possible control-flow paths to explore next: iterative deepening breadth-first algorithm
- maintains a "depth map" associating retreating control-flow edges with the number of times they have been crossed
- schedules using a priority queue
  - order retreating edges by depth
  - otherwise use a topological order

Analysis Scheduler: Join

- joins different executions to the same control point as eagerly as possible
  - DAGify the tree of executions
- joining depth maps turns out to be tricky
  - pointwise max: unsound due to potentially missing a short counter-example execution
  - pointwise min: unfair due to potentially exploring long executions before a shorter counter-example execution
  - so need to be a bit less eager

Symbolic State

- Symbolic representation of sets of states
- Assertions of a "byte blocks" theory of separation logic

$$
\begin{array}{rcll}
\mathcal{X} \ni x & ::= & \mathtt{x} \mid \cdots & \text{variables} \\
\mathcal{N} \ni \eta & ::= & 0 \mid 1 \mid \cdots & \text{numerals} \\
\mathcal{C} \ni f & ::= & \eta \mid \cdots & \text{constants} \\
A, B, N, L, E & ::= & x \mid f(E^*) \mid \forall x{:}\tau.\ B \mid \exists x{:}\tau.\ B & \text{terms} \\
Q & ::= & B \mid Q \vee Q \mid \exists x{:}\tau.\ Q & \text{formulas} \\
& & \mid \mathsf{emp} \mid Q * Q \mid L \xmapsto{[L;N)} A &
\end{array}
$$

First-Order Base

- assume some sorts
  - bool: classical Booleans
  - siz: object sizes
  - loc: memory locations
  - byt: bytes
- assume function symbols for standard operations

  $\simeq$, true, $\neg$, $\wedge$, $\vee$, $<$, $+$, $-$, $\eta\times$, 0,...
- some sorts $\mathcal{W}$ are "scalars"
  - contains at least siz and loc
  - sizeof : $\mathcal{W} \to \mathcal{N}$ size of scalar sort's representation

Interpretation:

- $\mathbb{U}_\sigma$ subset of universe interpreting sort σ
- $[\![f : \left(\bigtimes_{i=1}^{n} \sigma_i\right) \to \sigma]\!] \in \left(\bigtimes_{i=1}^{n} \mathbb{U}_{\sigma_i}\right) \to \mathbb{U}_\sigma$
  interpretation of each constant $f : \left(\bigtimes_{i=1}^{n} \sigma_i\right) \to \sigma$

Byte-Arrays

Simplified peek ahead:

$$\{L \mapsto \_\} \qquad \{L \mapsto \alpha\}$$

$$\texttt{store}\eta\ L\ E \qquad x \coloneqq \texttt{load}\eta\ L$$

$$\{L \mapsto \texttt{cast}^{\text{typeof } E}_{\text{byt}[\eta]}\, E\} \qquad \{L \mapsto \alpha * (x \simeq \texttt{cast}^{\text{byt}[\eta]}_{\text{typeof } x}\, \alpha)\}$$

Known size: $\text{byt}[\eta]$ for $\eta \in \mathcal{N}$

- $\texttt{cast}^{\omega}_{\text{byt}[\eta]}\, E : \omega \to \text{byt}[\eta]$
  reinterpret scalar ω as aggregate $\text{byt}[\eta]$ for $\eta = \text{sizeof}\, \omega$
- $\texttt{cast}^{\text{byt}[\eta]}_{\omega}\, A : \text{byt}[\eta] \to \omega$
  reinterpret aggregate $\text{byt}[\eta]$ as scalar ω for $\eta = \text{sizeof}\, \omega$
- $\texttt{cast}^{\text{byt}[\eta]}_{\omega}(\texttt{cast}^{\omega}_{\text{byt}[\eta]}\, E) \simeq E$ and vice versa

Byte-Arrays

Simplified peek ahead:

$$\{L \mapsto \_\}\,\texttt{memset}\ L\ E\ N\,\{L \mapsto E^N\}$$

N.b.: size of access to memory not always known

Unknown size: byt[·]
Size-tagged: byt[]

- $\langle N, A \rangle$: siz × byt[·] → byt[] tag a byte-array with its size
- E^N : byt × siz → byt[] iterated byte concatenation
- $A^\frown A$: byt[] × byt[] → byt[] concatenation
- $A[N;N)$: byt[] × siz × siz → byt[] extraction

Allocations

Simplified peek ahead:

$$\{\mathsf{emp}\} \qquad\qquad \{L \simeq 0 \vee L \overset{[L;N)}{\longmapsto} \langle N, \alpha \rangle\}$$
$$x \coloneqq \mathtt{alloc}\ N \qquad\qquad \mathtt{free}\ L$$
$$\{x \simeq 0 \vee \exists \alpha.\, x \overset{[x;N)}{\longmapsto} \langle N, \alpha \rangle\} \qquad\qquad \{\mathsf{emp}\}$$

N.b.: need to free the whole allocation

$L \overset{[B;M)}{\longmapsto} A$ keeps track of the enclosing allocation $[B;M)$ of the claimed chunk of memory

Block $[b;m)$

- m contiguous locations starting from b
- is an element (b, m) of $\mathbb{U}_{\mathsf{loc}} \times \mathbb{U}_{\mathsf{siz}}$
- often punned with interval $\{\, l \mid b \leqslant l \wedge l < b + m \,\}$

Heaps

Heaps:

$$\mathbb{H} = \left\{ h = (h_\alpha, h_\mathsf{b}) \in \begin{array}{r} \bigcup_{L \subset_\mathsf{fin} \mathbb{U}_\mathsf{loc}} \\ (L \to \mathbb{U}_\mathsf{byt}) \\ \times\, (\mathbb{U}_\mathsf{loc} \times \mathbb{U}_\mathsf{siz}) \end{array} \;\middle|\; \begin{array}{l} h_\mathsf{b} \text{ is compatible} \\ h_\mathsf{b} \text{ is inhabited over } L \\ L \text{ is enclosed in } h_\mathsf{b} \end{array} \right\}$$

Heap combination:

$$\_ \uplus \_ \;:\; \mathbb{H} \times \mathbb{H} \to \mathbb{H}_\bot$$

$$h \uplus h' = \begin{cases} \bot & \text{if } \operatorname{dom} h_\alpha \cap \operatorname{dom} h'_\alpha \neq \emptyset \text{ or } h_\mathsf{b} \frown h'_\mathsf{b} \\ (h_\alpha \cup h'_\alpha, h_\mathsf{b} \cup h'_\mathsf{b}) & \text{otherwise} \end{cases}$$

Semantics:

$$s, h \vDash L \xmapsto{[B;M)} A \text{ iff let } l, b, m, \alpha = [\![L]\!]_\mathbb{E}\, s,\ [\![B]\!]_\mathbb{E}\, s,\ [\![M]\!]_\mathbb{E}\, s,\ [\![A]\!]_\mathbb{E}\, s$$
$$[l;|\alpha|) \subseteq [b;m) \text{ and } (|\alpha| \simeq 0 \text{ implies } m \simeq 0) \text{ and}$$
$$h = \left(\left[[l;|\alpha|) \to \alpha \right], \{\, [b;m) \,\} \right)$$

Example (non-)Entailments

$$l \overset{[b;m)}{\longmapsto} \langle n, \alpha_0 \rangle * l{+}n \overset{[b;m)}{\longmapsto} \langle o, \alpha_1 \rangle \quad \vDash \quad l \overset{[b;m)}{\longmapsto} \langle n, \alpha_0 \rangle^\frown \langle o, \alpha_1 \rangle$$

$$l \overset{[b;m)}{\longmapsto} \langle n, \alpha_0 \rangle^\frown \langle o, \alpha_1 \rangle * n > 0 \quad \vDash \quad l \overset{[b;m)}{\longmapsto} \langle n, \alpha_0 \rangle * l{+}n \overset{[b;m)}{\longmapsto} \langle o, \alpha_1 \rangle$$

$$l \overset{[b;m)}{\longmapsto} \langle 0, \alpha_0 \rangle^\frown \langle n, \alpha_1 \rangle \quad \nvDash \quad l \overset{[b;m)}{\longmapsto} \langle 0, \alpha_0 \rangle * l \overset{[b;m)}{\longmapsto} \langle n, \alpha_1 \rangle$$

$$l \overset{[l;0)}{\longmapsto} 0^0 * l \overset{[l;0)}{\longmapsto} 0^0 \quad \vDash \quad \mathsf{false}$$

$$l \overset{[l;0)}{\longmapsto} 0^0 \quad \nvDash \quad \mathsf{emp} \qquad\qquad \mathsf{emp} \quad \nvDash \quad l \overset{[l;0)}{\longmapsto} 0^0$$

$$l \overset{[b;m)}{\longmapsto} \langle 0, \alpha \rangle \quad \vDash \quad l \simeq b * m \simeq 0 * l \overset{[b;m)}{\longmapsto} \langle 0, \alpha \rangle$$

$$l \overset{[b;0)}{\longmapsto} \langle n, \alpha \rangle \quad \vDash \quad l \simeq b * n \simeq 0 * l \overset{[b;0)}{\longmapsto} \langle n, \alpha \rangle$$

Symbolic Execution

Symbolic execution (simple):

$$\frac{\vec{u},\vec{x}' \mid P' \vDash \exists\vec{g}.\, F * R \qquad \overline{\vec{v} \mid \{F\}\ I\ \{Q\}}}{\vec{u} \mid \{\exists\vec{x}.\, P\}\ I\ \{\exists\vec{x}',\vec{g}.\, Q * R\}} \quad \begin{array}{l} \_' \text{ freshens } \vec{x} \text{ wrt } \vec{u},\vec{v} \\ \vec{g} = \vec{v} \setminus \vec{u} \end{array}$$

Derivable using frame, consequence, substitution, quantification

Excision judgment $\forall\vec{u}.\, C \ast M \mathrel{\backslash\!\!-} \exists\vec{x}.\, S \ast R$

- *valid* iff $C * M \vDash \exists\vec{x}.\, C * S * R$ valid
- *strong* only if every model of R can be extended with a model of $C * S$ to obtain a model of $C * M$
- *query* $\forall\vec{u}.\, M \mathrel{\backslash\!\!-} \exists\vec{x}.\, S$ yields R s.t. $\forall\vec{u}.\, \mathrm{emp} \ast M \mathrel{\backslash\!\!-} \exists\vec{x}.\, S \ast R$

Solver for excision queries yields proof procedure for symbolic execution

Symbolic Execution with Modified Variables

Actual small axioms:

$$\theta.\{\mathsf{emp}\} \quad \vec{x} \coloneqq \vec{E} \quad \{\ast\, x_i \simeq E_i\theta\}$$

$$\theta.\{L \overset{[b;m)}{\longmapsto} \langle \eta, \alpha\rangle\} \quad x \coloneqq \mathtt{load}_\eta\ L \quad \{(L \overset{[b;m)}{\longmapsto} \langle \eta, \alpha\rangle)\theta * x \simeq (\mathtt{cast}^{\mathsf{byt}[\eta]}_{\mathsf{typeof}\ x}\ \alpha)\theta\}$$

θ records renaming of modified variables:

$$\theta.\{P\}\ I\ \{Q\} \quad \text{iff} \quad \left\{\left(\ast_{o \in \mathrm{dom}\,\theta}\ o \simeq o\theta\right) * P\right\}\ I\ \left\{Q\right\}$$

Symbolic execution:

$$\frac{\forall \vec{u}, \vec{x}'.\, \mathsf{emp} \mathrel{\not\ast} P' \mathrel{\not\vdash} \exists \vec{g}.\, F \mathrel{\not\ast} R \qquad \overline{\vec{v} \mid \theta.\{F\}\ I\ \{Q\}}}{\vec{u} \mid \{\exists \vec{x}.\, P\}\ I\ \{\exists \vec{x}', \vec{g}.\, Q * \exists \vec{m}.\, R\theta\}}$$

θ freshens $(\mathrm{mod}\, I \cap (\mathrm{fv}\, F \cup \mathrm{fv}\, I))$ wrt $\vec{u}, \mathrm{fv}\, F$ $\quad \vec{g} = \vec{v} \setminus \vec{u}$

$\_'$ freshens $\vec{x}$ wrt $\vec{u}, \vec{v}$ $\quad \vec{m} = \mathrm{mod}\, I \setminus \mathrm{dom}\,\theta$

Frame Inference Solver

Primitive rules:

$$\frac{\forall \vec{u}, \vec{x}.\ S \vdash \mathsf{emp} \qquad \llcorner \vdash \exists \vec{x}.\ S}{\forall \vec{u}.\ C \mathrel{\ast\!\!\!\vdash} M \mathrel{\not\vdash} \exists \vec{x}.\ S \mathrel{\not\ast\!\!\!\vdash} M * S} \qquad \frac{\llcorner \vdash \mathsf{false}}{\forall \vec{u}.\ C \mathrel{\ast\!\!\!\vdash} M \mathrel{\not\vdash} \exists \vec{x}.\ S \mathrel{\not\ast\!\!\!\vdash} \mathsf{false}}$$

$$\frac{\forall \vec{u}.\ C * H \mathrel{\ast\!\!\!\vdash} M \mathrel{\not\vdash} \exists \vec{x}.\ S \mathrel{\not\ast\!\!\!\vdash} R}{\forall \vec{u}.\ C \mathrel{\ast\!\!\!\vdash} H * M \mathrel{\not\vdash} \exists \vec{x}.\ H * S \mathrel{\not\ast\!\!\!\vdash} R}$$

Pure consequences, in rule with conclusion $\forall \vec{u}.\ C \mathrel{\ast\!\!\!\vdash} M \mathrel{\not\vdash} \exists \vec{x}.\ S \mathrel{\not\ast\!\!\!\vdash} M$:

- $\llcorner = \forall \vec{u}.\ \lceil C * M \rceil$
- $\llcorner\!\!\lrcorner = \forall \vec{u}.\ \lceil C * M \rceil \wedge \lceil S \rceil$

where first-order term $\lceil Q \rceil : \mathsf{bool}$:

- *over-approximates* Q: $Q \vDash \lceil Q \rceil$
- better *equisatisfiable*: $[\![\lceil Q \rceil]\!]_{\mathrm{Q}} = \emptyset$ iff $[\![Q]\!]_{\mathrm{Q}} = \emptyset$
- optimal *exact*: $[\![\lceil Q \rceil]\!]_{\mathrm{E}} \sim \{\, s \mid s, h \vDash Q \,\}$ for some bijection $\sim$

Frame Inference Solver: $\mapsto$

Omitted:

- treatment of disjunction
- algorithm to choose left and right subformulas to try to match

Focus:

- theory-specific reasoning rules

Simple case:

$$\frac{\begin{array}{cc} & \forall \vec{u}.\, C * k \xmapsto{[b;m)} \langle o, \alpha \rangle \mathrel{⫽} M \\ \sqsubset \vdash k \simeq l * o \simeq n & \models \exists \vec{x}.\, b \simeq b' * m \simeq m' * \alpha \simeq \alpha' * S \mathrel{⫽} R \end{array}}{\forall \vec{u}.\, C \mathrel{⫽} k \xmapsto{[b;m)} \langle o, \alpha \rangle * M \models \exists \vec{x}.\, l \xmapsto{[b';m')} \langle n, \alpha' \rangle * S \mathrel{⫽} R}$$

Involves only equality reasoning and "subformula shuffling"

Frame Inference Solver: $\mapsto$

$$\frac{\begin{array}{l}\lfloor\!\rfloor \vdash l < k * k{+}o \simeq l{+}n \\ \quad \forall \vec{u}.\, C * k \xmapsto{[b;m)} \langle o, \alpha\rangle \nVdash M \\ \quad \vdash \exists \vec{x}, \alpha_0'.\, \langle k{-}l, \alpha_0'\rangle^\frown \langle o, \alpha\rangle \simeq \langle n, \alpha'\rangle * \\ \qquad l \xmapsto{[b';m')} \langle k{-}l, \alpha_0'\rangle * b \simeq b' * m \simeq m' * S \nVdash R\end{array}}{\forall \vec{u}.\, C \nVdash k \xmapsto{[b;m)} \langle o, \alpha\rangle * M \vdash \exists \vec{x}.\, l \xmapsto{[b';m')} \langle n, \alpha'\rangle * S \nVdash \exists \alpha_0'.\, R}$$

$$\frac{\begin{array}{l}\lfloor\!\rfloor \vdash k < l * k{+}o \simeq l{+}n \\ \quad \forall \vec{u}, \alpha_0, \alpha_1.\, C * l \xmapsto{[b;m)} \langle n, \alpha_1\rangle \\ \quad \nVdash k \xmapsto{[b;m)} \langle l{-}k, \alpha_0\rangle * \langle o, \alpha\rangle \simeq \langle l{-}k, \alpha_0\rangle^\frown \langle n, \alpha_1\rangle * M \\ \quad \vdash \exists \vec{x}.\, b \simeq b' * m \simeq m' * \alpha_1 \simeq \alpha' * S \nVdash R\end{array}}{\forall \vec{u}.\, C \nVdash k \xmapsto{[b;m)} \langle o, \alpha\rangle * M \vdash \exists \vec{x}.\, l \xmapsto{[b';m')} \langle n, \alpha'\rangle * S \nVdash \exists \alpha_0, \alpha_1.\, R}$$

Frame Inference Solver $\leftrightarrow$ First-Order Solver

First-order reasoning needed for frame inference:

- $\pi \vdash \_ \simeq \_$: equality queries (integers, byte-arrays, ...)
- $\pi \vdash \_ < \_$: inequality queries (integers)
- $\pi \vdash \mathsf{false}$: FO inconsistency checking
- $\pi \vdash \exists \vec{x}.\, S$: FO entailment

Canonizing First-Order Theory Solver: Interface

"Shostak-style" first-order theory solver:

- σ: "solution set" of equations maintained by solver
- $\texttt{assert}\,(a \simeq b)\,\sigma$: conjoin $a \simeq b$ to σ, yield $\bot$ if unsat or σ' ow
- $a\sigma$: canonical form of term a under equations σ

Answer queries:

- $P \vdash \mathsf{false}$: $\texttt{assert}\,P\,\emptyset = \bot$
- $P \vdash a \simeq b$: $\text{let } \sigma = \texttt{assert}\,P\,\emptyset \text{ in } a\sigma = b\sigma$
- $P \vdash a < b$: $\text{let } \sigma = \texttt{assert}\,P\,\emptyset \text{ in } (b-a)\sigma \text{ is a positive integer}$
- $P \vdash b$: $\text{let } \sigma = \texttt{assert}\,P\,\emptyset \text{ in } b\sigma = \mathsf{true}$

May need to handle Boolean combinations in P: either in frame inference solver or directly on σ (perhaps inefficiently).

Canonizing First-Order Theory Solver: Canonizer

η is a *canonizer* if:

- $\vDash a \simeq b$ iff $\eta a = \eta b$
- $fv(\eta a) \subseteq \mathrm{fv}(a)$
- $\eta b = b$ for each subterm b of ηa

For linear arithmetic, rewriting into ordered sum of monomials is a canonizer.
E.g.:

$$\eta\,(x + z + w - y - w + z) = x - y + 2z$$

Canonizing First-Order Theory Solver: Solver

`solve` is a *solver* if:

- `solve` $(a \simeq b) = \bot$ iff $a \simeq b$ is unsat
- `solve` $(a \simeq b) = \sigma$ for σ a solution substitution/set s.t.:
  - $\operatorname{dom} \sigma \subseteq \operatorname{fv}(a \simeq b)$
  - σ is idempotent
  - σ and $a \simeq b$ are logically equivalent

For linear arithmetic, solving for one variable gives a solver.
E.g.:

$$\texttt{solve}\ (2x - 3y + 2z = 5y) = [x \mapsto 4y - z]$$

Byte-Array Canonizer

Rewrite system generated by:

$$\langle n, \alpha \rangle \Rightarrow \alpha \text{ when } n \equiv |\alpha|$$

$$\_[\_;0) \Rightarrow \langle\rangle$$

$$\alpha[m;k)[o;l) \Rightarrow \alpha[m+o;l) \text{ when } k \geqslant o+l$$

$$e^n[o;l) \Rightarrow e^l \text{ when } n \geqslant o+l$$

$$\langle n, \alpha \rangle [0;n) \Rightarrow \langle n, \alpha \rangle$$

$$(\alpha_0 \cdots \alpha_i \cdots \alpha_j \cdots)[|\alpha_0 \cdots \alpha_{i-1}|; |\alpha_i \cdots \alpha_j|) \Rightarrow \alpha_i \cdots \alpha_j$$

$$\alpha^\frown(\beta^\frown\gamma) \Rightarrow \alpha^\frown\beta^\frown\gamma$$

$$\langle n, \alpha \rangle [o;k)^\frown \langle n, \alpha \rangle [o+k;l) \Rightarrow \langle n, \alpha \rangle [o;k+l) \text{ when } n \geqslant o+k+l$$

$$e^{m\frown}e^n \Rightarrow e^{m+n}$$

Byte-Array Solver

$$\langle 0, \alpha \rangle \simeq \beta \Rightarrow \alpha \simeq \langle\rangle \wedge \beta \simeq \langle\rangle \qquad x \simeq \langle n, \alpha \rangle \Rightarrow x \simeq \alpha$$

$$\langle n, \alpha \rangle \simeq \langle m, \beta \rangle \Rightarrow n \simeq m \wedge \alpha \simeq \beta \qquad \langle n, \alpha \rangle \simeq \beta \Rightarrow n \simeq |\beta| \wedge \alpha \simeq \beta$$

$$x \simeq \alpha[o;l) \Rightarrow x \mapsto \alpha[o;l) \text{ when } x \notin \text{fv}\, \alpha[o;l)$$

$$x \simeq \alpha[o;l) \Rightarrow \alpha[o;l) \mapsto \langle l, x \rangle \text{ when } x \in \text{fv}\, \alpha[o;l)$$

$$x \simeq \alpha_0 \cdots \alpha_n \Rightarrow x \mapsto \alpha_0 \cdots \alpha_n \text{ when } x \notin \text{fv}\, \alpha_0 \cdots \alpha_n$$

$$x \simeq \alpha_0 \cdots \alpha_n \Rightarrow \langle |\alpha_0 \cdots \alpha_n|, x \rangle \simeq \alpha_0 \cdots \alpha_n \text{ when } x \in \text{fv}\, \alpha_0 \cdots \alpha_n$$

$$\alpha_0 \cdots \alpha_i \cdots \alpha_n \simeq \beta \Rightarrow |\alpha_0 \cdots \alpha_n| \simeq |\beta| \wedge \cdots \wedge \alpha_i \simeq \beta\,[|\alpha_0 \cdots \alpha_{i-1}|;\, |\alpha_i|) \wedge \cdots$$

$$\alpha[o;l) \simeq \beta \Rightarrow l \simeq |\beta| \wedge \alpha \simeq \langle |\alpha|, \chi \rangle [0;o) \frown \beta \frown \langle |\alpha|, \chi \rangle [o+l;|\alpha|-o-l) \quad \text{where } \chi \text{ fresh}$$

Witnessing Existentials

Recall:

$$\frac{\forall \vec{u}, \vec{x}.\ S \vdash \mathsf{emp} \qquad \leftthreetimes \vdash \exists \vec{x}.\ S}{\forall \vec{u}.\ C \mathrel{-\!\!*} M \mathrel{\reflectbox{$\vdash$}} \exists \vec{x}.\ S \mathrel{\reflectbox{$-\!\!*$}} M * S}$$

Use FO solver's solution set for $\leftthreetimes$ to compute σ

- if $X\sigma = U$ then $\mathrm{fv}\, X \cap \vec{x} \neq \emptyset$ and $\mathrm{fv}\, U \cap \vec{x} = \emptyset$
- enumerating congruence classes
- solving polynomial equations for existential variables
- solving for byte-arrays by concatenating extractions:

$$\alpha \simeq \langle m, x \rangle[0;n) \ \wedge\ \beta \simeq \langle m, x \rangle[n;m-n) \quad \models \quad x \simeq \alpha \frown \beta$$

$$\frac{\forall \vec{u}, \vec{k}.\ C \mathrel{-\!\!*} M * \sigma_m \mathrel{\reflectbox{$\vdash$}} \exists \vec{x} \setminus \vec{k}.\ \sigma_s * S\sigma \mathrel{\reflectbox{$-\!\!*$}} R}{\forall \vec{u}.\ C \mathrel{-\!\!*} M \mathrel{\reflectbox{$\vdash$}} \exists \vec{x}.\ S \mathrel{\reflectbox{$-\!\!*$}} R}$$

$$\leftthreetimes \models \sigma \qquad\qquad \vec{k} = \vec{x} \setminus \mathrm{fv}\, S\sigma$$

$\sigma_m \wedge \sigma_s = \sigma$ where σ_m is (maximal) s.t. $(C * M) \models \exists \vec{k}.\ \sigma_m$

Eliminating Existentials

- Performed regularly during symbolic execution / analysis
- Rewrites formulas $\exists \vec{x}.\, S$ to $\exists \vec{x} \setminus \vec{k}.\, \sigma_s * S\sigma$ similarly to solver
- Operates directly on NNF
- Existential quantification used for scope management
  - prevent constraints on overwritten values of variables from interfering with constraints on their current values (Floyd's assignment axiom and cousins)
  - logical analogue of shadowing variables when pushing a stack frame
  - out-scoping local variables when popping stack frames
  - procedure parameter passing
- "Garbage collects" constraints about execution history

Parting Thoughts

- LLVM makes C++ approachable
- Fuzzer harnesses lower overhead of applying tools to existing code
- Analyzing bitcode requires more arithmetic support than might otherwise be strictly necessary
- Something like the theory for byte-arrays to manipulate memory contents is needed, seems to work, simpler would be better
- Interaction between Shostak-style first-order solvers and separation logic proof search works nicely
- Redundant quantifier elimination is mandatory